

Metrics Of Positive Scalar Curvature And Generalised Morse Functions

THE SPACE OF METRICS OF POSITIVE SCALAR CURVATURE

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ABSTRACT. We prove non-triviality of higher homotopy groups of the space of positive scalar curvature metrics on high dimensional spheres and other spin manifolds. In contrast to previous approaches, our construction yields families of such metrics that remain homotopically non-trivial in the (obscure) moduli space of metrics of positive scalar curvature. A key result is the construction of smooth fiber bundles over spheres with non-vanishing \hat{A} -genus. These are the first examples with simply connected base which exhibit non-multiplicativity of the \hat{A} -genus in fiber bundles: they provide examples where the \hat{A} -genus leads to a non-trivial homomorphism on π_k of a diffeomorphism group of a closed manifold.

1. INTRODUCTION AND SUMMARY

The classification of positive scalar curvature metrics on closed smooth manifolds is a central topic in Riemannian geometry. Whereas the existence question could be resolved in many cases and is governed by the (stable) Gromov-Lawson conjecture (compare e.g. [L7]), information on the topological complexity of the space of positive scalar curvature metrics on a given manifold M has been sparse and only recently some progress could be made [F7].

We denote by $\text{Riem}^+(M)$ the space of Riemannian metrics of positive scalar curvature on M equipped with the C^∞ -topology. If it is not empty, we want to give information on the homotopy groups $\pi_k(\text{Riem}^+(M), g)$ for $g \in \text{Riem}^+(M)$ in the different path components of $\text{Riem}^+(M)$. One method to construct non-zero elements in these homotopy groups, developed by Hitchin, is to pull back g along a family of diffeomorphisms of M . In [F7, Theorem 4.7] this was used to prove existence of non-zero classes of order two in $\pi_1(\text{Riem}^+(M))$ for certain manifolds M . In [F7, Corollary 1.5] this method has been refined to show that there exist non-zero elements of order two in infinitely many degrees of $\pi_k(\text{Riem}^+(M), g)$, when M is a spin manifold admitting a metric g of positive scalar curvature.

In the paper at hand we construct elements of infinite order in $\pi_k(\text{Riem}^+(M))$ for $k \in \mathbb{N}$. Our construction is quite different from Hitchin's in that it is not connected to topological properties of the diffeomorphism group of M .

The following is the first main result of our paper.

Theorem 1.1. *Let $k \geq 0$ be a natural number. Then there is a natural number $N(k)$ with the following property: For each $n \geq N(k)$ and each spin manifold M admitting a metric g of positive scalar curvature and of dimension $\dim M = k + 1$, the homotopy group*

$$\pi_k(\text{Riem}^+(M), g)$$

contains elements of infinite order if $k \geq 1$, and infinitely many different elements if $k = 0$.

For $k = 0$ this statement is well known (with $N(k) = 1$), see [F7, Theorem IV.7.7] and [F7, Theorem 4.47] for the special case $M = S^{4n-1}$.

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Abstract: It is well known that isotopic metrics of positive scalar curvature are concordant. Whether or not the converse holds is an open question, at least in dimensions ≥ 5 . Topics covered include the scalar curvature, surgery theorem and Gromov-Lawson cobordisms. In general, little is known about the topology of $\text{Riem}^+(X)$, although some results are known. Metrics of positive scalar curvature and generalised Morse functions, part II. This provides a direct way to construct metrics of positive scalar curvature and generalised Morse functions, Part II. Article. Metrics of Positive Scalar Curvature and Generalised Morse Functions, Part I. M Walsh. *Memoirs of the American Mathematical Society*, 80, 24*, The connection with generalised Morse functions and Part II xiv Isotopy and concordance in the space of metrics of positive scalar curvature Warped Riemannian metric whose scalar curvature function is strictly positive is a problem which has been extensively studied. It is well known that isotopic metrics of positive scalar curvature are concordant. Whether or not the converse holds is an open question, at least in dimensions ≥ 5 . Then M carries a metric of positive scalar curvature if and only if M admits a metric of positive scalar curvature and generalised Morse functions, part I.

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